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An Exponential Lower Bound for a Restricted Class of Monotone Formulae for 2-Unsatisfiability

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by

David A. Plaisted

September 1978

DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN · URBANA, ILLINOIS

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David A. Plaisted

Department of Computer Science
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1. Introduction

The complexity of minimal Boolean circuits for computing Boolean functions is polynomially related to a lower bound on the Turing complexity of the function [3]. Proving lower bounds on the Boolean circuit complexity of various functions is one potential method of obtaining lower bounds on their Turing complexity. As a small step in this direction, we prove an exponential lower bound on the monotone Boolean formula complexity of a specific function, if the formulae are restricted in certain ways. The function we consider is the satisfiability of a set of 2-literal clauses. Each clause is represented as an input to the Boolean formula. An input of TRUE indicates that the clause is in the set of clauses being considered, and FALSE indicated the clause is not in the set of clauses. We want the formula to have a value of TRUE if the set of clauses is inconsistent and FALSE if it is consistent. Although satisfiability of sets of 2-literal clauses can be decided in polynomial time [1], so that polynomial size circuits exist, the results presented here may suggest approaches for obtaining lower bounds for the Boolean circuit complexity of 3-satisfiability.

2. The lower Bound

Definition: A Boolean formula (or expression) over the propositional variables $x_1, \ldots, x_k$ is a well-formed expression composed of the variables $x_1, \ldots, x_k$ together with the unary connective $\neg$, the binary connectives $\wedge$ and $\vee$, and the constants TRUE and FALSE. The connectives $\neg$, $\wedge$, $\vee$ represent negation, conjunction, and disjunction, respectively. The value of a Boolean expression, when the values of $x_1 \ldots x_k$ are given, is determined in the usual way. We write $\bar{x}_i$ as an abbreviation for $\neg x_i$.

Definition: A Boolean function is a mapping from $\{\text{TRUE, FALSE}\}^n$ into {TRUE, FALSE} for some integer n.

Definition: If E is a Boolean expression over the variables $x_1, \ldots, x_k$ then the function computed by E is the Boolean function $f(x_1, \ldots, x_k)$ whose value is the value of E when the values of $x_1, \ldots, x_k$ are specified. We write f_E for the function computed by E.

Definition: Two Boolean expressions E1 and E2 are equivalent if $f_{E1} = f_{E2}$.

Definition: A Boolean formula E is monotone if the only connectives in E are $\wedge$ and $\vee$.

Definition: Suppose f is a Boolean function of the propositional variables $x_1, x_2, \ldots, x_k$. Suppose sets $a_1, a_2, \ldots, a_m$ are subsets of $\{x_1, x_2, \ldots, x_k\} \cup \{\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k\}$. We say $a_1 \wedge a_2 \wedge \ldots \wedge a_m$ is a conjunctive normal form of f if all of the following are true:

1. $f(x_1, \ldots, x_k)$ is true iff for all i, $1 \leq i \leq m$, either there exists j such that x_j is TRUE and $x_j \in a_i$ or there exists j such that x_j is FALSE and $\bar{x}_j \in a_i$.
2. There does not exist i, $1 \leq i \leq m$, such that $x_j \in a_i$ and $\bar{x}_j \in a_i$ for some j.
3. There do not exist i, j, $1 \leq i, j \leq m$, $i \neq j$, such that $a_i \subseteq a_j$.

Thus $\{x_1, x_2\} \wedge \{\bar{x}_1, x_3\}$ is a conjunctive normal form of $(x_1 \vee x_2) \wedge (\bar{x}_1 \vee x_3)$. Note that each a_j represents the disjunction of its elements.

If E is a Boolean formula, a conjunctive normal form of E is defined to be a conjunctive normal form of the function computed by E. If E is monotone, we require all elements of the a_j to be (uncomplemented) propositional variables. (Expressions of the form $\bar{x}_i$ are called complemented propositional variables.) If E is monotone, then its conjunctive normal form is unique (up to permutations of the a_j).

Definition: If the conjunctive normal form of E is $a_1 \wedge \ldots \wedge a_n$, then cnf(E) is the set $\{a_1, a_2, \ldots, a_n\}$. We call elements of cnf(E) disjunctions of cnf(E). Sometimes we write cnf(E) as $a_1 \wedge a_2 \wedge \ldots \wedge a_n$.

Definition: An interpretation I of a set $\{P_1, \ldots, P_n\}$ of predicate symbols is an assignment of truth-values TRUE or FALSE to the predicate symbols. Thus there are 2^n interpretations of $\{P_1, \ldots, P_n\}$. We say P_i is true in I if I assigns the value TRUE to P_i and P_i is false in I otherwise.

Definition: A literal over $\{P_1, \ldots, P_n\}$ is an element of the set $\{P_1, P_2, \ldots, P_n\} \cup \{\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_n\}$.

Definition: A literal L is true in interpretation I if L is of the form P_i and P_i is true in I or if L is of the form $\bar{P}_i$ and P_i is false in I.

Definition: A _clause_ C over $\{P_1, \ldots, P_n\}$ is a set of literals over $\{P_1, \ldots, P_n\}$. A clause represents the disjunction of its elements. Thus C is true in interpretation I if at least one literal of C is true in I; otherwise C is false in I.

Example: Let I be the interpretation making all predicate symbols true. Then $\{P_1, \bar{P}_2, \bar{P}_3\}$ is true in I but $\{\bar{P}_1, \bar{P}_2, \bar{P}_3\}$ is false in I.

Definition: The _complement_ $\bar{L}$ of a literal L over $\{P_1, \ldots, P_n\}$ is defined as follows:

If L is of the form P_i then $\bar{L}$ is $\bar{P}_i$.

If L is of the form $\bar{P}_i$ then $\bar{L}$ is P_i.

Definition: Let $\mathcal{I}_n$ be the set of interpretations of $\{P_1, \ldots, P_n\}$. For $I \in \mathcal{I}_n$, let Contr(I) be the set of 2-literal clauses C over $\{P_1, \ldots, P_n\}$ such that C is _false_ in I.

From now on we consider 2-literal clauses over $\{P_1, \ldots, P_n\}$ to be propositional variables. An assignment of truth-values to the variables represents a set of clauses in the following way:

> If the clause C is assigned the value TRUE, then C is in the set of clauses, otherwise C is not in the set of clauses.

Boolean functions of these $\binom{2n}{2}$ variables represent properties of sets of clauses.

Definition: A set S of clauses over $\{P_1, \ldots, P_n\}$ is _inconsistent_ (or _unsatisfiable_) iff for all $I \in \mathcal{I}_n$, there exists $C \in S$ such that C is false in I.

Note that the conjunctive normal form of the inconsistency function is

$$\bigwedge_{I \in \mathcal{I}_n} \text{Contr}(I).$$

Example: The set {C1, C2, C3} of clauses is inconsistent, where C1 = $\{P_1\}$, C2 = $\{\bar{P}_1, P_2\}$ and C3 = $\{\bar{P}_1, \bar{P}_2\}$. However, any proper subset of {C1, C2, C3} is consistent.

We generally do not consider clauses C such that for some i, $P_i \varepsilon C$ and $\bar{P}_i \varepsilon C$. Such clauses are called tautologies and are true in all interpretations. There are only n 2-literal tautologies over $\{P_1, \ldots, P_n\}$. There are $4\binom{n}{2}$ or $2n^2 - 2n$ 2-literal non-tautologies over $\{P_1, \ldots, P_n\}$. We are thus considering Boolean functions of $2n^2 - 2n$ variables.

Definition: A monotone Boolean formula E has the subset property if for all sub-formulae E1 of E, the following is true:

Suppose $\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_m$ is the conjunctive normal form for E1. Suppose $\beta_1 \wedge \beta_2 \wedge \ldots \wedge \beta_n$ is the conjunctive normal form for E. Then for all i, $1 \leq i \leq m$, there exists j, $1 \leq j \leq n$ such that $\alpha_i \subseteq \beta_j$.

Definition: Suppose E is a monotone Boolean formula. We say E satisfies the replacement condition if for all sub-formulae E1 of E, the following is true:

Let E' be E with an occurrence of E1 replaced by FALSE. Then there exist disjunctions α and α' of cnf(E) and cnf(E'), respectively, such that α' is a proper subset of α.

Theorem 1. Let E be a monotone Boolean formula over the $\binom{2n}{2}-n$ propositional variables representing 2-literal clauses over $\{P_1, \ldots, P_n\}$ which are not tautologies. Suppose that E represents the inconsistency property of sets of 2-literal clauses over $\{P_1, \ldots, P_n\}$. (That is, E is true iff the set of clauses represented by the values of its inputs, is inconsistent.) Suppose that E satisfies the replacement condition and has the subset property. Then E has at least $2^n - 1$ "$\wedge$" connectives.

An example of such an expression E for 1-literal clauses might be $(\{P_1\} \wedge \{\bar{P}_1\}) \vee (\{P_2\} \wedge \{\bar{P}_2\})$. This expression represents the set of clauses containing both $\{P_1\}$ and $\{\bar{P}_1\}$ or both $\{P_2\}$ and $\{\bar{P}_2\}$. This expression is monotone and logically implies inconsistency, but is not equivalent to inconsistency if $n > 2$.

Fact: Suppose E is a monotone Boolean formula of the form $E1 \wedge E2$. Then cnf(E) consists of the minimal elements of $\text{cnf}(E1) \cup \text{cnf}(E2)$ in the subset ordering.

Suppose E is a monotone Boolean formula of the form $E1 \vee E2$. Then cnf(E) consists of the minimal elements of $\{\mathcal{a} \cup \mathcal{B}: \mathcal{a} \in \text{cnf}(E1), \mathcal{B} \in \text{cnf}(E2)\}$ in the subset ordering.

Definition: Suppose E is a monotone Boolean formula. Then Interp(E) is $\{I \in \mathcal{I}_n: \exists \mathcal{a} \in \text{cnf}(E) \text{ such that } \mathcal{a} \subseteq \text{Contr}(I)\}$.

Fact: Suppose E is a monotone Boolean formula of the form $E1 \wedge E2$. Then $\text{Interp}(E) = \text{Interp}(E1) \cup \text{Interp}(E2)$.

Fact: Suppose E is a monotone Boolean formula of the form $E1 \vee E2$. Then $\text{Interp}(E) = \text{Interp}(E1) \cap \text{Interp}(E2)$.

Definition: Suppose $\mathcal{a} \subseteq \text{Contr}(I)$ for $I \in \mathcal{I}_n$. Suppose $\mathcal{L}$ is a set of literals that are false in I. We write $\mathcal{a} \underset{\mathcal{L}}{\sim} I$ if the following is true:

For all literals L such that L is false in I, there exists a clause C in $\mathcal{a}$ such that $L \in C$. Also, if L1 and L2 are literals in $\mathcal{L}$, there must exist a clause C1 in $\mathcal{a}$ such that $L1 \in C1$ and $L2 \in C1$. (The set $\mathcal{L}$ need not include all the literals that are false in I.)

Note that if $\mathcal{a} \underset{\mathcal{L}}{\sim} I$ for some $\mathcal{L}$ then $\cup\{C: C \in \mathcal{a}\}$ is the set of literals that are false in I. Also, if there exists $I \in \mathcal{I}_n$ such that $\mathcal{a} \underset{\mathcal{L}}{\sim} I$ then I is unique.

Lemma 1. Suppose $\mathcal{a} \underset{\mathcal{L}}{\sim} I$ for some interpretation $I \in \mathcal{I}_n$. Suppose $\mathcal{B}_1 \subseteq \text{Contr}(I1)$ and $\mathcal{B}_2 \subseteq \text{Contr}(I2)$ for some interpretations $I1, I2 \in \mathcal{I}_n$. Suppose that for all literals L such that L is true in I1 but false in I2 or false in I1 but true in I2, either $L \in \mathcal{L}$ or $\bar{L} \in \mathcal{L}$. Suppose that $\mathcal{a} \subseteq \mathcal{B}_1 \cup \mathcal{B}_2$. Then either $I = I1$ or $I = I2$.

Proof: Not difficult.

Lemma 2. Suppose E is a monotone Boolean formula which satisfies the replacement condition. Suppose E1 is a sub-formula of E. Then E1 satisfies the replacement condition also.

Proof: By induction on the depth of E. If E is a propositional variable, the result is immediate. Let E' be E with some sub-formula of E1 replaced by FALSE. Let E1' be E1 with this sub-formula replaced by FALSE. We know that some disjunction of cnf(E') is a proper subset of some disjunction of cnf(E). Suppose E is of the form F1 $\wedge$ F2. Suppose E1 is a sub-formula of F1. Let F1' be F1 with E1 replaced by E1'. Then some disjunction of cnf(F1') must be a proper subset of some disjunction of cnf(F1). Hence F1 satisfies the replacement condition. By induction, E1 does too.

A similar argument applies if E is of the form F1 $\vee$ F2.

Definition: Suppose E and E1 are monotone Boolean formulae. Then we say a propositional variable x is *diminished from* E *to* E1 if there exist disjunctions a and $\mathcal{B}$ of cnf(E) and cnf(E1), respectively, such that $\mathcal{B} \subset a$ and $x \varepsilon a - \mathcal{B}$.

Lemma 3. Suppose E is a monotone Boolean formula satisfying the replacement condition. Suppose E is of the form F1 $\vee$ F2. Suppose x is a propositional variable of F1. Then x must be diminished from E to F2.

Proof: Let F1' be F1 with some occurrence of x replaced by FALSE. Let E' be E with this occurrence of x replaced by FALSE. Let $a \varepsilon$ cnf(E), $a' \varepsilon$ cnf(E') be disjunctions such that a' is a proper subset of a. Then there exist disjunctions $\mathcal{B}_1' \varepsilon$ F1', $\mathcal{B}_2 \varepsilon$ F2 such that $\mathcal{B}_1' \cup \mathcal{B}_2 = a'$. We cannot have $\mathcal{B}_1' \varepsilon$ cnf(E) since $a' \not\varepsilon$ cnf(E). Hence $x \not\varepsilon a'$ and there must exist $\mathcal{B}_1 \varepsilon$ cnf(E1) such that $x \varepsilon \mathcal{B}_1$ and $\mathcal{B}_1' = \mathcal{B}_1 - \{x\}$. Since $x \not\varepsilon a'$, $x \not\varepsilon \mathcal{B}_2$. Also, a' is a proper subset of a, hence $x \varepsilon a$ and $\mathcal{B}_1 \cup \mathcal{B}_2$ is a subset of a. Hence $a = \mathcal{B}_1 \cup \mathcal{B}_2$ so $a = a' \cup \{x\}$. Hence x is diminished from E to F2, since $\mathcal{B}_2 \subset a$ and $x \varepsilon a - \mathcal{B}_2$.

Definition: Suppose E is a monotone Boolean formula such that for all $a \in \text{cnf}(E)$, there exists $I \in \mathcal{I}_n$ such that $a \subseteq \text{Contr}(I)$. Then we say E has the contradiction property.

Definition: Two clauses C1 and C2 are resolvable if there exists a literal L such that $L \in C1$ and $\bar{L} \in C2$.

Definition: Let $\mathcal{E}$ be the class of monotone Boolean formulae F satisfying the following conditions:

1. F satisfies the replacement condition.
2. Suppose $a_1, a_2 \in \text{cnf}(F)$ and $I1, I2 \in \mathcal{I}_n$. Suppose $a_1 \subseteq \text{Contr}(I1)$ and $a_2 \subseteq \text{Contr}(I2)$. Suppose C1 and C2 are two clauses such that C1 and C2 are resolvable and such that $C1 \in \text{Contr}(I1)$ and $C2 \in \text{Contr}(I2)$. Then $C1 \in a_1$ and $C2 \in a_2$.
3. F and all its sub-formulae have the contradiction property.

Lemma 4. Suppose E is a Boolean formula in the class $\mathcal{E}$. Then

a) If E is of the form $E1 \vee E2$, then at least one of E1 and E2 are in the class $\mathcal{E}$.

b) If E is of the form $E1 \wedge E2$, then both of E1 and E2 are in the class $\mathcal{E}$.

Proof: Part b) is easy. We prove part a). Suppose $E \in \mathcal{E}$ and E is of the form $E1 \vee E2$. Suppose $a_1, a_2 \in \text{cnf}(E)$ and $I1, I2 \in \mathcal{I}_n$. Suppose $a_1 \subseteq \text{Contr}(I1)$ and $a_2 \subseteq \text{Contr}(I2)$. Suppose $C1 \in \text{Contr}(I1)$ and $C2 \in \text{Contr}(I2)$. Suppose C1 and C2 are resolvable. In particular, suppose $L \in C1$ and $\bar{L} \in C2$ for literal L.

We know by definition of E that C1 ε a_1 and C2 ε a_2. Since E satisfies the replacement condition, C1 must be diminished from E to either E1 or E2. Similarly for C2.

Suppose C1 is not diminished from E to E1. We claim that C2 is not diminished from E to E1.

Assume this is not true. Let B_1 be an element of cnf(E1) such that $B_1 \subseteq$ Contr(I1) and such that C1 ε B_1. Let B_2 be an element of cnf(E1) such that $B_2 \subseteq$ Contr(I2) and C2 $\not\varepsilon$ B_2. Let D_1 be an element of cnf(E2) such that $D_1 \subseteq$ Contr(I1) and C1 $\not\varepsilon$ D_1. (We know that C1 ε B_1 since C1 is not diminished from E to E1. Also, B_2 exists because C2 is diminished from E to E1. Furthermore, D_1 exists because C1 is diminished from E to E2).

Now, $B_2 \cup D_1$ must have some element of cnf(E) as a subset, since E is E1 $\vee$ E2. Suppose a ε cnf(E) and $a \subseteq B_2 \cup D_1$. Let I ε I_n be such that $a \subseteq$ Contr(I). Let L be the set of literals that are false in I but whose truth-values in I1 and I2 differ. It is not difficult to show that $a \overset{\sim}{_L}$I using the fact that E is in the class E. Hence by Lemma 1 we have the I = I1 or I = I2. However, we cannot have I = I1 since C1 $\not\varepsilon$ D_1 and C1 $\not\varepsilon$ B_2. We cannot have I = I2 since C2 $\not\varepsilon$ B_2 and C2 $\not\varepsilon$ B_1. Thus we have a contradiction.

Hence for C1 and C2 as given, if C1 is not diminished from E to E1, neither is C2.

Let D_1 and D_2 be elements of cnf(E). Suppose $D_1 \subseteq$ Contr(J1) and $D_2 \subseteq$ Contr(J2) for J1, J2 ε I_n. Let M be a literal such that M is true in J1 and false in J2. Recall that L is a literal such that L ε C1 and $\bar{L}$ ε C2. Now, the clause {L, M} or the clause {$\bar{L}$, M} must occur in some disjunction of cnf(E) by definition of E. Suppose

{L, M} occurs in some disjunction of cnf(E). Then {L, M} cannot be diminished from E to E1, by reasoning as above with clauses C2 and {L, M}. Some clause containing $\bar{M}$ must occur in some disjunction of cnf(E), by definition of the class *E*. But no such clause can be diminished from E to E1, by reasoning as above. Extending this argument, no clause containing M can be diminished from E to E1.

It follows that E1 ε *E* also. If C1 is not diminished from E to E2, then E2 ε *E* by similar reasoning. We now show that C1 cannot be diminished from E to both E1 and E2, and so either E1 ε *E* or E2 ε *E*.

Suppose C1 is diminished from E to both E1 and E2. Then there exist a_1' ε cnf(E1), a_2' ε cnf(E2) and interpretations I1', I2' ε I_n having the following properties:

1. $a_1' \subseteq$ Contr(I1') and $a_2' \subseteq$ Contr(I2').
2. C1 ε Contr(I1') and C1 ε Contr(I2').
3. C1 $\not\varepsilon$ a_1' and C1 $\not\varepsilon$ a_2'.
4. For some a ε cnf(E), $a \subseteq a_1' \cup a_2'$.

Suppose $a \subseteq$ Contr(I); then C1 ε Contr(I). This is because U{C: C ε a} must include all literals that are false in I, hence U{C: C ε a_1' or C ε a_2'} includes all these literals, hence all literals of C1 must occur in U{C: C ε a_1' or C ε a_2'}. However, no complement of a literal in C1 can occur in U{C: C ε a_1'} or in U{C: C ε a_2'}.

Since C1 $\not\varepsilon$ a_1' and C1 $\not\varepsilon$ a_2', therefore C1 $\not\varepsilon$ a. But this contradicts the definition of *E*, since C1 and C2 are resolvable and C2 ε a_2 for a_2 ε cnf(E).

Therefore C1 cannot be diminished from E to both E1 and E2. Hence E1 ε E or E2 ε E, and the theorem is proven.

Theorem 2. Suppose E is in the class E of Boolean formulae. Let N(E) be |Interp(E)|, the number of interpretations I ε I_n such that there exists a ε cnf(E) such that $a \subseteq$ Contr(I). Then E has at least N(E)-1 "$\wedge$" connectives.

Proof: By induction on the depth of the formula. Suppose E is of the form E1 $\vee$ E2. Then either E1 ε E or E2 ε E. Also, Interps(E) = Interps(E1) $\cap$ Interps(E2). Hence N(E) $\geq$ N(E1) and N(E) $\geq$ N(E2). If E1 is in E, we can assume inductively that E1 (hence E) has at least N(E1) "$\wedge$" connectives, and similarly for E2.

Suppose E is of the form E1 $\wedge$ E2. Then E1 ε E and E2 ε E. Also, Interps(E) = Interps(E1) $\cup$ Interps(E2) so N(E) $\leq$ N(E1) + N(E2). We can assume inductively that E1 has at least N(E1) - 1 "$\wedge$" connectives and that E2 has at least N(E2)-1 "$\wedge$" connectives. Hence E has at least N(E1) + N(E2)-1 "$\wedge$" connectives, which is not less than N(E)-1 "$\wedge$" connectives.

Suppose E is a propositional variable. Then either N(E)=1, in which case the result is immediate, or else E $\not\varepsilon$ E.

This completes the proof.

Corollary (Theorem 1): Suppose E is a monotone Boolean formula for inconsistency of 2-literal clauses over the predicate symbols $\{P_1, P_2, \ldots, P_n\}$. Suppose E has the subset property and satisfies the replacement condition. Then E has at least $2^n - 1$ "$\wedge$" connectives.

Proof: E is in the class E, and N(E) = 2^n.

Note that the number of propositional variables in E is $4\binom{n}{2}$, which is $O(n^2)$. Thus we have an exponential lower bound for this restricted class of formulae.

A similar proof will work for 3-literal clauses, 4-literal clauses, et cetera. However, we know that inconsistency of 2-literal clauses can be decided in polynomial time, and so polynomial size Boolean circuits exist for inconsistency of 2-literal clauses. If such polynomial size circuits exist for 3-literal clauses, then P = NP relative to a slowly utilized oracle [2]. This seems unlikely. In fact, if no such polynomial size circuits exist for 3-literal clauses, then P ≠ NP. In order to prove P ≠ NP, we have to remove the subset and replacement conditions, allow negations, and transfer the results to circuits instead of formulae. This would require a method of distinguishing between the 2-literal and the 3-literal case.

3. Conclusions

We have shown an exponential lower bound on the complexity of a monotone Boolean formula for 2-unsatisfiability, under certain restrictions on the structure of the formulae. The argument is a top-down combinatorial argument. It is conceivable that extensions of this result to the Boolean formula complexity of 3-unsatisfiability might prove P ≠ NP.

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